# MA261 Quiz 9

July 26, 2016

**Problem 1.**

Use Green's theorem to evaluate the line integral along the given positively oriented curve $C$.

$$\int_C (2x + y - y^3)dx + (2y + x + x^3)dy$$

where $C$ is the circle of radius 2 centered at the origin.

*Solution.*

$$P(x, y) = 2x + y - y^3$$
$$Q(x, y) = 2y + x + x^3$$
$$\int_C P(x, y)dx + Q(x, y)dy = \int_{x^2+y^2\leq 4} \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} dA$$
$$= \int_{x^2+y^2\leq 4} (1 + 3x^2) - (1 - 3y^2)dA$$
$$= \int_0^{2\pi} \int_0^2 3r^3 dr d\theta = 24\pi$$

**Problem 2.**

Find the curl of the given vector field

$$F(x, y, z) = \langle xy + yz, yz + xz, xyz + xy \rangle$$

*Solution.*

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy + yz & yz + xz & xyz + xy \end{vmatrix}$$

$$= (xz - y)\mathbf{i} + (-yz)\mathbf{j} + (-x)\mathbf{k}$$